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THE DISTRIBUTION OF THE NUMBER OF TARGETS THAT  
WILL BE RANDOMLY ENGAGED BY A GROUP OF WEAPONS

ARTHUR GROVES

MAY 1981

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## CONTENTS

	Page
1. INTRODUCTION . . . . .	5
2. DISCUSSION . . . . .	5
2.1 Assumptions and Problem Statement . . . . .	5
2.2 Method of Solution . . . . .	5
3. CONCLUSIONS AND RECOMMENDATIONS . . . . .	7
APPENDIX A - DERIVATION OF RECURSIVE EXPRESSION FOR $B(m,x)$ . .	17
APPENDIX B - PROOF THAT $\sum_{x=1}^n P(x,m,n) = 1$ . . . . .	21
DISTRIBUTION LIST . . . . .	27

# THE DISTRIBUTION OF THE NUMBER OF TARGETS THAT WILL BE RANDOMLY ENGAGED BY A GROUP OF WEAPONS

## 1. INTRODUCTION

In war-gaming and other weapons analysis studies, it is frequently assumed that available firepower is evenly distributed among available targets. This assumption presumes a degree of coordination among the weapons that may not exist on the battlefield. It may be that each weapon will choose a target at random from among the set of targets. This type of target selection leads to the possibility that some targets receive fire from many weapons, while others receive no fire at all. This report derives the probability distribution for the number of targets engaged under the assumption of random target selection by each weapon.

## 2. DISCUSSION

In the following sections, the assumptions and statement of the engagement problem are formalized and the solution is developed.

### 2.1 Assumptions and Problem Statement.

The following assumptions will govern the derivation of the distribution of the number of targets engaged:

- There are  $m$  weapons selecting targets.
- There are  $n$  targets in the group from which each weapon selects a target.
- Each weapon selects one and only one of the targets to engage.
- A target may be engaged by more than one weapon.
- Each weapon is equally likely to engage each target.

Under these assumptions, the problem is to find:

$P(x,m,n)$  = the probability that exactly  $x$  targets are engaged,  
and

$\bar{x}(m,n)$  = the expected or average number of targets engaged.

### 2.2 Method of Solution.

The problem will be treated as a counting problem. Let  $N(x,m,n)$  denote the number of ways in which exactly  $x$  targets can be selected at random by  $m$  weapons from among  $n$  targets. Further, let  $N(m,n)$  denote the total number of ways in which  $m$  weapons can select targets from among  $n$  targets.

Then

$$P(x,m,n) = \frac{N(x,m,n)}{N(m,n)}. \quad (1)$$

First, consider the denominator of (1). Since there are  $n$  targets, each weapon can select its target in  $n$  ways. There are  $m$  weapons, so

$$N(m,n) = \underbrace{(n)(n)(n)\dots\dots(n)}_{m \text{ factors}} = n^m$$

Now consider the numerator of (1), which can be expressed as the product of two factors as follows:

$$N(x,m,n) = A(x,n) \times B(m,x) \quad (2)$$

where  $A(x,n)$  denotes the number of ways that exactly  $x$  targets can be selected at random from among  $n$  targets, and  $B(m,x)$  denotes the number of ways that  $m$  weapons can divide their fire among  $x$  targets. Clearly,  $A(x,n) = \binom{n}{x}$ , the number of combinations of  $n$  things taken  $x$  at a time. An expression for  $B(m,x)$  is not as easily obtained. In "Introduction to Combinatorial Mathematics," by Liu, McGraw-Hill, 1968, Page 38, it is shown that

$$B(m,x) = \sum_{i=0}^x (-1)^i \binom{x}{i} (x-i)^m \quad (3)$$

A more intuitive expression for  $B(m,x)$  is derived in Appendix A. This is a recursive definition of  $B(m,x)$ , as follows

$$B(m,x) = \sum_{i=1}^{m-x+1} \binom{m}{i} B(m-i,x-1) \quad (4)$$

with starting values  $B(k,1) = 1$  for all  $k$ . Using Equation (3) to define  $B(m,x)$ , it follows from Equation (2) that:

$$N(x,m,n) = \binom{n}{x} \sum_{i=0}^x (-1)^i \binom{x}{i} (x-i)^m \quad (5)$$

and

$$P(x,m,n) = \frac{\binom{n}{x}}{n^m} \sum_{i=0}^x (-1)^i \binom{x}{i} (x-i)^m, \quad (6)$$

which is the desired result. All that remains is to carefully specify the domain of this probability density junction.

There are certain values of  $x$  relative to  $m$  and  $n$  for which this probability is clearly equal to zero. These values are

$x > m$  (One cannot engage more targets than he has weapons to engage them.)

and  $x > n$  (One cannot engage more targets than there are targets available.)

Thus, the complete specification of the desired probability is

$$P(x, m, n) = \begin{cases} 0 & \text{if } x > \min(m, n) \\ \frac{\binom{n}{x}}{n^m} \sum_{i=0}^x (-1)^i \binom{x}{i} (x-i)^m & \text{if } x \leq \min(m, n) \end{cases} \quad (7)$$

Appendix B presents a proof that this result satisfies the requirement of a probability density function that

$$\sum_{x=1}^n P(x, m, n) = 1.$$

The expected value of  $x$  (the average number of targets engaged) is found by definition to be

$$\bar{x}(m, n) = \sum_{x=1}^n x P(x, m, n) \quad (8)$$

While the formulas for  $P(x, m, n)$  and  $\bar{x}(m, n)$  are somewhat unwieldy for hand computation, they are well suited to high speed digital computation at least for reasonably small values of  $m$  and  $n$ . For larger  $m$  and  $n$ , care must be taken, for the size of some of the partial results [ $B(m, x)$  for example] can easily exceed the word length in a machine.

Tables 1 through 10 give values of  $P(x, m, n)$  for  $m = 1, 2, \dots, 10$  respectively, for each value of  $n$  between 1 and 10.

Table 11 and Figure 1 give values of  $\bar{x}(m, n)$  for the same range of values of  $m$  and  $n$ .

### 3. CONCLUSIONS AND RECOMMENDATIONS

A model which assumes an even distribution of available fire among available targets tends to overestimate the attrition to that set of targets. This can best be seen by noting that there is a diminishing incremental effect as additional shots are fired at a given target. Thus, a given increment of kill probability is more effective in terms of expected effect on the set of targets, the fewer the total number of such increments

that are applied to that same target. For example, given two weapons and two targets, with each weapon having a single shot kill probability of 0.6 against a target, the expected number of kills is

$$(1)(.6) + (1)(.6) = 1.2$$

if each weapon fires at its own target, but is

$$(1)[1-(1-.6)^2] + (1)(0) = 0.84$$

if both weapons fire at the same target, leaving the second target un-attacked.

This effect could be contributing to the observed effect that most wargame models overestimate the effectiveness of weapons as compared to battle experience.

It is recommended that consideration be given to incorporating the distributions derived in this report into pertinent combat models, at least to the extent of allowing available fire to be divided among the expected number of targets engaged (Equation (11) or Figure 1) rather than assuming it is divided equally among all available targets.

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TABLE 1 PROBABILITY THAT EXACTLY X TARGETS ARE ENGAGED

m = NUMBER OF WEAPONS = 1

x	TOTAL NUMBER OF AVAILABLE TARGETS (n)									
	1	2	3	4	5	6	7	8	9	10
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

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TABLE 2 PROBABILITY THAT EXACTLY X TARGETS ARE ENGAGED

$m = \text{NUMBER OF WEAPONS} = 2$

TOTAL NUMBER OF AVAILABLE TARGETS (n)										
x	1	2	3	4	5	6	7	8	9	10
1	1.000	0.500	0.333	0.250	0.200	0.167	0.143	0.125	0.111	0.100
2	-	0.500	0.667	0.750	0.800	0.833	0.857	0.875	0.889	0.900

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TABLE 3 PROBABILITY THAT EXACTLY X TARGETS ARE ENGAGED

$m = \text{NUMBER OF WEAPONS} = 3$

TOTAL NUMBER OF AVAILABLE TARGETS (n)										
x	1	2	3	4	5	6	7	8	9	10
1	1.000	0.250	0.111	0.063	0.040	0.028	0.020	0.016	0.012	0.010
2	-	0.750	0.667	0.563	0.480	0.417	0.367	0.328	0.296	0.270
3	-	-	0.222	0.375	0.480	0.556	0.612	0.656	0.691	0.720

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TABLE 4 PROBABILITY THAT EXACTLY X TARGETS ARE ENGAGED

m = NUMBER OF WEAPONS = 4

TOTAL NUMBER OF AVAILABLE TARGETS (n)

x	1	2	3	4	5	6	7	8	9	10
1	1.000	0.125	0.037	0.016	0.008	0.005	0.003	0.002	0.001	0.001
2	-	0.875	0.519	0.328	0.224	0.162	0.122	0.096	0.077	0.063
3	-	-	0.444	0.563	0.576	0.556	0.525	0.492	0.461	0.432
4	-	-	-	0.094	0.192	0.278	0.350	0.410	0.461	0.504

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TABLE 5 PROBABILITY THAT EXACTLY X TARGETS ARE ENGAGED

m = NUMBER OF WEAPONS = 5

TOTAL NUMBER OF AVAILABLE TARGETS (n)

x	1	2	3	4	5	6	7	8	9	10
1	1.000	0.063	0.012	0.004	0.002	0.001	0.000	0.000	0.000	0.000
2	-	0.938	0.370	0.176	0.096	0.058	0.037	0.026	0.018	0.014
3	-	-	0.617	0.586	0.480	0.386	0.312	0.256	0.213	0.180
4	-	-	-	0.234	0.384	0.463	0.500	0.513	0.512	0.504
5	-	-	-	-	0.038	0.093	0.150	0.205	0.256	0.302

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TABLE 6 PROBABILITY THAT EXACTLY X TARGETS ARE ENGAGED

m = NUMBER OF WEAPONS = 6

TOTAL NUMBER OF AVAILABLE TARGETS (n)

x	1	2	3	4	5	6	7	8	9	10
1	1.000	0.031	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000
2	-	0.969	0.255	0.091	0.040	0.020	0.011	0.007	0.004	0.003
3	-	-	0.741	0.527	0.346	0.231	0.161	0.115	0.085	0.065
4	-	-	-	0.381	0.499	0.502	0.464	0.417	0.370	0.328
5	-	-	-	-	0.115	0.231	0.321	0.385	0.427	0.454
6	-	-	-	-	-	0.015	0.043	0.077	0.114	0.151

TABLE 7 PROBABILITY THAT EXACTLY X TARGETS ARE ENGAGED

m = NUMBER OF WEAPONS = 7

TOTAL NUMBER OF AVAILABLE TARGETS (n)

x	1	2	3	4	5	6	7	8	9	10
1	1.000	0.016	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	-	0.984	0.173	0.046	0.016	0.007	0.003	0.002	0.001	0.001
3	-	-	0.826	0.441	0.231	0.129	0.077	0.048	0.032	0.022
4	-	-	-	0.513	0.538	0.450	0.357	0.280	0.221	0.176
5	-	-	-	-	0.215	0.360	0.428	0.449	0.443	0.423
6	-	-	-	-	-	0.054	0.129	0.202	0.266	0.318
7	-	-	-	-	-	-	0.006	0.019	0.038	0.060

TABLE 8 PROBABILITY THAT EXACTLY X TARGETS ARE ENGAGED

m = NUMBER OF WEAPONS = 8

TOTAL NUMBER OF AVAILABLE TARGETS (n)

x	1	2	3	4	5	6	7	8	9	10
1	1.000	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	-	0.992	0.116	0.023	0.007	0.002	0.001	0.000	0.000	0.000
3	-	-	0.883	0.354	0.148	0.069	0.035	0.019	0.011	0.007
4	-	-	-	0.623	0.523	0.365	0.248	0.170	0.119	0.086
5	-	-	-	-	0.323	0.450	0.459	0.421	0.369	0.318
6	-	-	-	-	-	0.114	0.233	0.320	0.374	0.402
7	-	-	-	-	-	-	0.024	0.067	0.118	0.169
8	-	-	-	-	-	-	-	0.002	0.008	0.018

TABLE 9 PROBABILITY THAT EXACTLY X TARGETS ARE ENGAGED

m = NUMBER OF WEAPONS = 9

TOTAL NUMBER OF AVAILABLE TARGETS (n)

x	1	2	3	4	5	6	7	8	9	10
1	1.000	0.004	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	-	0.996	0.078	0.012	0.003	0.001	0.000	0.000	0.000	0.000
3	-	-	0.922	0.277	0.093	0.036	0.016	0.008	0.004	0.002
4	-	-	-	0.711	0.477	0.278	0.162	0.097	0.061	0.039
5	-	-	-	-	0.427	0.497	0.434	0.348	0.271	0.210
6	-	-	-	-	-	0.189	0.330	0.397	0.413	0.400
7	-	-	-	-	-	-	0.058	0.139	0.216	0.279
8	-	-	-	-	-	-	-	0.011	0.034	0.065
9	-	-	-	-	-	-	-	-	0.001	0.004

---

TABLE 10 PROBABILITY THAT EXACTLY X TARGETS ARE ENGAGED

$m = \text{NUMBER OF WEAPONS} = 10$

TOTAL NUMBER OF AVAILABLE TARGETS ( $n$ )

x	1	2	3	4	5	6	7	8	9	10
1	1.000	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	-	0.998	0.052	0.006	0.001	0.000	0.000	0.000	0.000	0.000
3	-	-	0.948	0.214	0.057	0.019	0.007	0.003	0.001	0.001
4	-	-	-	0.781	0.419	0.203	0.101	0.053	0.030	0.017
5	-	-	-	-	0.523	0.506	0.379	0.266	0.184	0.129
6	-	-	-	-	-	0.272	0.407	0.429	0.396	0.345
7	-	-	-	-	-	-	0.105	0.221	0.306	0.356
8	-	-	-	-	-	-	-	0.028	0.078	0.136
9	-	-	-	-	-	-	-	-	0.005	0.016
10	-	-	-	-	-	-	-	-	-	0.000

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TABLE 11 EXPECTED NUMBER OF TARGETS ENGAGED

TOTAL NUMBER OF AVAILABLE TARGETS (n)

	1	2	3	4	5	6	7	8	9	10
1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2	1.00	1.50	1.67	1.75	1.80	1.83	1.86	1.88	1.89	1.90
3	1.00	1.75	2.11	2.31	2.44	2.53	2.59	2.64	2.68	2.71
4	1.00	1.88	2.41	2.73	2.95	3.11	3.22	3.31	3.38	3.44
5	1.00	1.94	2.60	3.05	3.36	3.59	3.76	3.90	4.01	4.10
6	1.00	1.97	2.74	3.29	3.69	3.99	4.22	4.41	4.56	4.69
7	1.00	1.98	2.82	3.47	3.95	4.33	4.62	4.86	5.05	5.22
8	1.00	1.99	2.88	3.60	4.16	4.60	4.96	5.25	5.49	5.70
9	1.00	2.00	2.92	3.70	4.33	4.84	5.25	5.59	5.88	6.13
10	1.00	2.00	2.95	3.77	4.46	5.03	5.50	5.90	6.23	6.51

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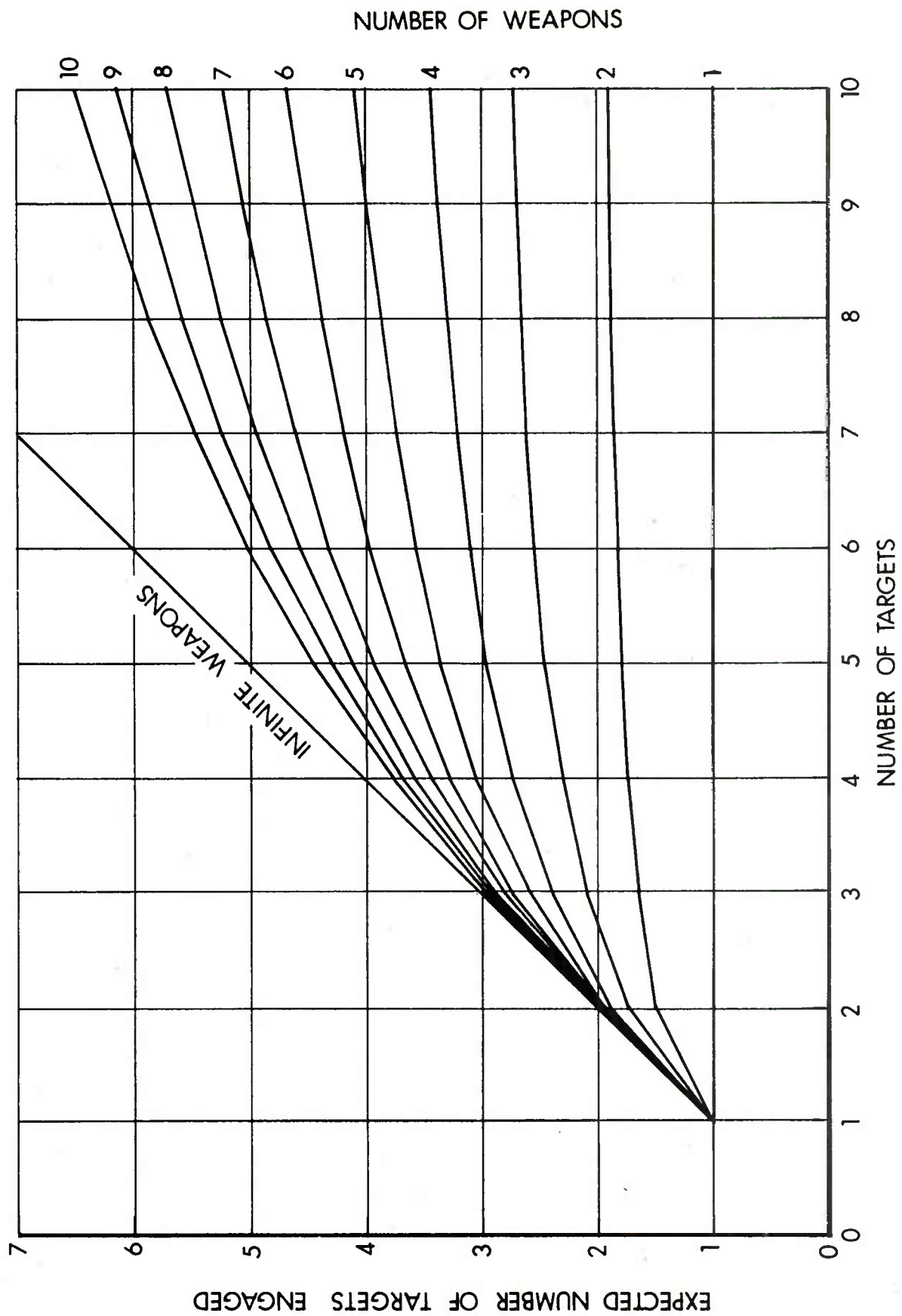


Figure 1. Expected Number of Targets Engaged if Each Weapon Selects a Target at Random.

## APPENDIX A

### DERIVATION OF RECURSIVE EXPRESSION FOR $B(m,x)$



APPENDIX A  
DERIVATION OF RECURSIVE EXPRESSION FOR  $B(m,x)$

$B(m,x)$  is defined to be the number of ways that  $m$  weapons can divide their fire among  $x$  targets, with each target receiving fire from at least one weapon.

Let  $i$  denote the number of weapons choosing to attack target #1. Since there are  $m$  weapons altogether, there are  $\binom{m}{i}$  ways in which this set of  $i$  weapons can be chosen. Once  $i$  weapons have been chosen, there are  $m-i$  weapons left, and  $x-1$  targets left to be attacked. The number of ways that  $m-i$  weapons can divide their fire among  $x-1$  targets is simply  $B(m-i, x-1)$ , using the definition of  $B$  given above.

Therefore, the number of ways in which  $m$  weapons can divide their fire among  $x$  targets, with exactly  $i$  weapons firing at target #1, is

$$\binom{m}{i} B(m-i, x-1)$$

Since various values of  $i$  are possible,

$$B(m,x) = \sum_i \binom{m}{i} B(m-i, x-1).$$

Since at least one weapon must fire at target #1, this summation starts at  $i=1$ . Since at least one weapon must be left to fire at each of the remaining  $x-1$  targets, the maximum allowable value of  $i$  is  $m-(x-1) = m-x+1$ . Thus,

$$B(m,x) = \sum_{i=1}^{m-x+1} \binom{m}{i} B(m-i, x-1)$$

This provides a recursive expression for  $B(m,x)$ . It only remains to provide a starting value for  $B$ . If  $x=1$ , that is, if there is only one target, there is only one way that fire can be divided - all the weapons must attack that target. Thus,

$$B(k,1) = 1 \quad \text{for all } k \geq 1$$

This provides the required starting values. Therefore, the complete specifications of the recursive expression is

$$B(k,1) = 1 \quad \text{for all } k \geq 1$$

and 
$$B(m,x) = \sum_{i=1}^{m-x+1} \binom{m}{i} B(m-i, x-1) \quad \text{for } 2 \leq x \leq m.$$

## APPENDIX B

PROOF THAT  $\sum_{x=1}^n P(x,m,n) = 1$

## APPENDIX B

Equation (6) gives the probability that exactly  $x$  targets are attacked to be

$$P(x, m, n) = \frac{\binom{n}{x}}{n^m} \sum_{i=0}^x (-1)^i \binom{x}{i} (x-i)^m$$

It will be shown that this function satisfies one requirement of a probability density function, namely that

$$\sum_{x=1}^n P(x, m, n) = 1.$$

That is, it must be shown that

$$J \equiv \sum_{x=1}^n \binom{n}{x} \sum_{i=0}^x (-1)^i \binom{x}{i} (x-i)^m = n^m$$

The first step is to rewrite  $J$  in the form

$$J = \sum_{i=1}^n C_i i^m$$

This is done by writing out  $J$  term by term and then collecting terms with like values of  $i^m$ .

$$x=1 \quad \binom{n}{1} \sum_{i=0}^1 (-1)^i \binom{1}{i} (1-i)^m = \binom{n}{1} \binom{1}{0} 1^m$$

$$x=2 \quad \binom{n}{2} \sum_{i=0}^2 (-1)^i \binom{2}{i} (2-i)^m = \binom{n}{2} \left[ \binom{2}{0} 2^m - \binom{2}{1} 1^m \right]$$

$$x=3 \quad \binom{n}{3} \sum_{i=0}^3 (-1)^i \binom{3}{i} (3-i)^m = \binom{n}{3} \left[ \binom{3}{0} 3^m - \binom{3}{1} 2^m + \binom{3}{2} 1^m \right]$$

$$x=4 \quad \binom{n}{4} \sum_{i=0}^4 (-1)^i \binom{4}{i} (4-i)^m = \binom{n}{4} \left[ \binom{4}{0} 4^m - \binom{4}{1} 3^m + \binom{4}{2} 2^m - \binom{4}{3} 1^m \right]$$

$$x=5 \quad \binom{n}{5} \sum_{i=0}^5 (-1)^i \binom{5}{i} (5-i)^m = \binom{n}{5} \left[ \binom{5}{0} 5^m - \binom{5}{1} 4^m + \binom{5}{2} 3^m - \binom{5}{3} 2^m + \binom{5}{4} 1^m \right]$$

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$$x=n \quad \binom{n}{n} \sum_{i=0}^n (-1)^i \binom{n}{i} (n-i)^m = \binom{n}{n} \left[ \binom{n}{0} n^m - \binom{n}{1} (n-1)^m + \binom{n}{2} (n-2)^m + \dots + (-1)^{n-1} \binom{n}{n-1} 1^m \right]$$

Collecting terms in like values of  $i^m$ , the coefficient of  $i^m$  is

$$C_i = \sum_{j=i}^n (-1)^{j-i} \binom{j}{j-i} \binom{n}{j}$$

By letting  $k=j-i$ , this coefficient becomes

$$C_i = \sum_{k=0}^{n-i} (-1)^k \binom{k+i}{k} \binom{n}{k+i}$$

Applying the definition  $\binom{a}{b} = \frac{a!}{b!(a-b)!}$ ,

$$C_i = \frac{n!}{i!} \sum_{k=0}^{n-i} (-1)^k \frac{1}{k!(n-k-i)!}$$

But  $n-i$  is fixed relative to this summation. Call it  $N$ . Therefore,

$$C_i = \frac{n!}{i!} \sum_{k=0}^N (-1)^k \frac{1}{k!(N-k)!}$$

$$= \frac{n!}{i!N!} \sum_{k=0}^N (-1)^k \frac{N!}{k!(N-k)!}$$

$$C_i = \binom{n}{i} \sum_{k=0}^N (-1)^k \binom{N}{k}$$

This summation can be evaluated by recognizing it as a special case of the binomial expansion of  $(a+b)^N$ . In general,

$$(a+b)^N = \sum_{k=0}^N \binom{N}{k} a^k b^{N-k}$$

For  $a = -1$  and  $b=1$ , we have

$$0^N = \sum_{k=0}^N (-1)^k \binom{N}{k}$$

But  $0^N=0$  for  $N>0$ . Therefore,  $C_i=0$  for  $N>0$ , that is, for  $i<n$ . For  $i=n$  (that is, for  $N=0$ ),

$$C_n = \binom{n}{n} \sum_{k=0}^0 (-1)^k \binom{0}{k} = \binom{n}{n} (-1)^0 \binom{0}{0} = 1$$

Thus,  $J=n^m$ , establishing the desired result.

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